AD-A038 325 BATTELLE COLUMBUS LABS OHIO F/G 17/8

OPTICAL PATTERN RECOGNITION OF DISTANT TARGETS: A NEW SYSTEM OF--ETC(U)

JAN 77 J R SHEWELL DAHC04-72-A-0001

UNCLASSIFIED RE-CR-77-3 NL

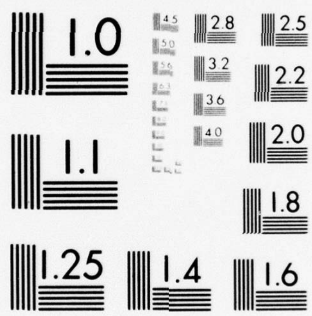
1 OF 1

ADA038325

END

DATE FILMED

5-77



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(12)
B.S.

TECHNICAL REPORT RE-CR-77-3

OPTICAL PATTERN RECOGNITION OF DISTANT TARGETS: A NEW SYSTEM OF OPTICAL SCALING AND MODELING

19 January 1977

Approved for public release; distribution unlimited.

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama

Prepared for:

Advanced Sensors Directorate
US Army Missile Research, Development and Engineering Laboratory
US Army Missile Command
Redstone Arsenal, Alabama 35809

DISPOSITION INSTRUCTIONS

DESTROY THIS REPORT WHEN IT IS NO LONGER NEEDED. DO NOT RETURN IT TO THE ORIGINATOR.

DISCLAIMER

THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.

TRADE NAMES

USE OF TRADE NAMES OR MANUFACTURERS IN THIS REPORT DOES NOT CONSTITUTE AN OFFICIAL INDORSEMENT OR APPROVAL OF THE USE OF SUCH COMMERCIAL HARDWARE OR SOFTWARE.

CONTENTS

		Page
I.	INTRODUCTION	3
II.	BACKGROUND	3
III.	GENERAL SCALING EXAMPLE	8
IV.	DEFINITIONS AND RULES OF ORDINARY SCALING	14
V.	TARGET ILLUMINATION	16
VI.	SCALING EQUATIONS FOR A ONE-LENS SYSTEM	18
VII.	SCALING EQUATIONS FOR A TWO-LENS SYSTEM	28
VIII.	GENERALIZATION TO AN n-LENS SYSTEM	36
IX.	TREATMENT OF A SPATIAL FILTER	37
X.	MODELING THE TACTICAL OPTICAL PATTERN RECOGNITION SYSTEM	40
XI.	DISCUSSION	42

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER (18) RE-CR-77-3 (18)	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) (6) OPTICAL PATTERN RECOGNITION OF DISTANT TARGETS: A NEW SYSTEM OF OPTICAL SCALING AND MODELING.	5. TYPE OF REPORT & PERIOD COVERED (9) Technical Report,
	6. ~~PERFORMING ORG.~~ REPORT NUMBER RE-CR-77-3
7. AUTHOR(s) (10) John R. Shewell ~~Consulting Physicist~~	8. CONTRACT OR GRANT NUMBER(s) Contract No. DAHC04-72-A-0001 ~~Task Order 76-84~~ (15)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Battelle Columbus Labs, Ohio per file - + contract Refs.	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Commander US Army Missile Command Attn: DRSMI-RPR Redstone Arsenal, Alabama 35809	12. REPORT DATE (11) 19 January 1977
	13. NUMBER OF PAGES 45 (12) 48p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Commander US Army Missile Command Attn: DRSMI-RE Redstone Arsenal, Alabama 35809	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Optical pattern recognition
Field-scale system
Target signatures
Optical scaling and modeling

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents the basic principles of a new system of optical scaling and modeling. These principles apply to any optical situation in which the illumination is totally coherent and for which the paraxial approximation is valid. The primary scaling relation is a geometrical similarity between the output diffraction patterns of an original optical system and its model. The optical scaling based on these principles is called ordinary scaling.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 407 080

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

I. INTRODUCTION

A preliminary system design for optical pattern recognition of distant targets was presented in a previous report. The design was for a field-scale system, and was based on certain concepts of target signatures and recognition criteria that are associated with field-scale properties. These signatures and recognition criteria are believed to be realistic and reasonable, but they have not been tested directly through actual observations and measurements.

Direct testing would require either field tests using an actual system based on the design that has been presented, or laboratory tests using a model system that is appropriately scaled in its optical performance. The construction of a full-scale system and the carrying out of field measurements would be expensive. It is the author's opinion that the present status of and need for optical pattern recognition techniques in tactical applications would not justify the expense.

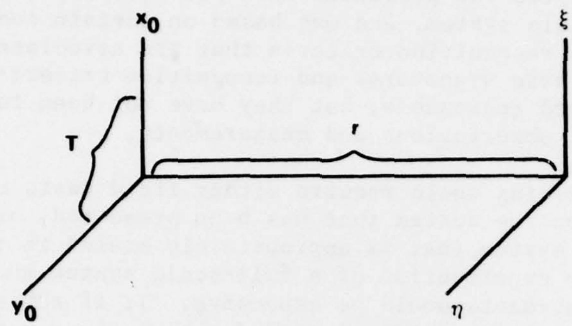
Laboratory tests using a model system would be less expensive, and the cost could probably be justified at this stage of tactical application of optical pattern recognition. Also, a model system can be modified with relative ease to sharpen recognition criteria and lead to improved design concepts for field-scale systems. Thus, a laboratory experiment with an appropriate model system is the logical step following the design exercise for the field-scale system.

For optical diffraction, there is a simple and well-known scaling law that depends only upon the ratio of the two different wavelengths used in the two different diffraction experiments. This law is applicable to any optical pattern recognition system. Unfortunately, it is of no practical value for the present case because realistic wavelength ratios are only approximately 0.1; and a 0.1 scale having a 10-km range would be 1 km, which would still require a field system and field tests.

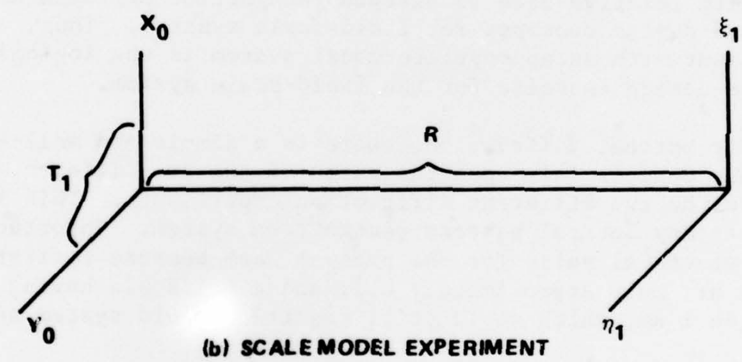
The development of a laboratory system that correctly models the optical performance of the field-scale system therefore requires a more general optical scaling law. Such a law has not previously been developed. Consequently, the author has developed a law. This report presents, in detail, the derivation of the new scaling law and the scaling or modeling equations that are suitable for the preliminary tactical system design.

II. BACKGROUND

Two diffraction experiments using the wavelengths λ and λ_1 are considered where one experiment is required to be a "scale model" of the other. For simplicity, the objects are assumed to be transparencies illuminated by plane waves, and only diffraction by free space are considered. The experiments are analyzed using the Fresnel-Kirchhoff integral. These systems are illustrated in Figure 1.



(a) ORIGINAL EXPERIMENT



(b) SCALE MODEL EXPERIMENT

Figure 1. Diffraction experiments.

In the original experiment, the amplitude in the receiving plane is given by

$$U(\xi_1\eta) = -\frac{A_0 i}{\lambda}\iint T(x_0\,,\,y_0)$$

$$\times \frac{e^{ik\left[(x_0-\xi)^2+(y_0-\eta)^2+r^2\right]^{1/2}}}{\left[(x_0-\xi)^2+(y_0-\eta)^2+r^2\right]^{1/2}} \quad ,$$

where

$$k = \frac{2\pi}{\lambda} \quad .$$

For purposes of scaling, one follows the convenient and usual practice of introducing dimensionless variables:

$$u_0 = kx_0 \ , \ v_0 = ky_0 \ , \ \overline{r} = kr \quad ,$$

$$\sigma = k\xi \ , \ \tau = k\eta \quad .$$

Then,

$$U(\xi_1\eta) = U\left(\frac{\sigma}{k} \ , \ \frac{\tau}{k}\right) = -\frac{A_0 i}{2\pi}\iint T\left(\frac{u_0}{k} \ , \ \frac{v_0}{k}\right)$$

$$\times \frac{e^{i\left[(u_0-\sigma)^2+(v_0-\tau)^2+\overline{r}^2\right]^{1/2}}}{\left[(u_0 - \sigma)^2 + (v_0 - \tau)^2 + \overline{r}^2\right]^{1/2}} du_0 dv_0 \quad .$$

If one also writes

$$V(\sigma \ , \ \tau) = U\left(\frac{\sigma}{k} \ , \ \frac{\tau}{k}\right)$$

and

$$\mathcal{T}(u_0 \ , \ v_0) = T\left(\frac{u_0}{k} \ , \ \frac{v_0}{k}\right) \quad ,$$

the completely nondimensionalized equation for the amplitude in the receiving plane is

$$V(\sigma \ , \ \tau) = -\frac{A_0 i}{2\pi}\iint \mathcal{T}(u_0 \ , \ v_0)$$

$$\times \frac{e^{i\left[(u_0-\sigma)^2+(v_0-\tau)^2+\overline{r}^2\right]^{1/2}}}{\left[(u_0 - \sigma) + (v_0 - \tau)^2 + \overline{r}^2\right]^{1/2}} du_0 dv_0 \quad .$$

For the model system, the amplitude in the receiving plane is

$$U_1(\xi_1 , \eta_1) = - \frac{A_1 i}{\lambda_1} \iint T_1(X_0 , Y_0)$$

$$\times \frac{e^{ik_1\left[(X_0-\xi_1)^2+(Y_0-\eta_1)^2+R^2\right]^{1/2}}}{\left[(X_0 - \xi_1)^2 + (Y_0 = \eta_1)^2 + R^2\right]^{1/2}} \quad .$$

Introducing the nondimensionalizing substitutions

$$\bar{u}_0 = k_1 X_0 \ , \ \bar{v}_0 = k_1 Y_0 \ , \ \bar{R} = k_1 R \quad ,$$

$$\sigma_1 = k_1 \xi_1 \ , \ \tau_1 = k_1 \eta_1 \quad ,$$

$$V_1(\sigma_1 , \tau_1) = U_1\left(\frac{\sigma_1}{k_1} , \frac{\tau_1}{k_1}\right) \quad ,$$

and

$$\mathcal{T}_1(\bar{u}_0 , \bar{v}_0) = T_1\left(\frac{\bar{u}_0}{k_1} , \frac{\bar{v}_0}{k_1}\right) \quad ,$$

one finds that

$$V_1(\sigma_1 , \tau_1) = -\frac{A_1 i}{2\pi} \iint \mathcal{T}_1(\bar{u}_0 , \bar{v}_0)$$

$$\times \frac{e^{i\left[(\bar{u}_0-\sigma_1)^2+(\bar{v}_0-\tau_1)^2+\bar{R}^2\right]^{1/2}}}{\left[(\bar{u}_0 - \sigma_1)^2 + (\bar{v}_0 - \tau_1)^2 + \bar{R}^2\right]^{1/2}} \, d\bar{u}_0 d\bar{v}_0 \quad .$$

It is seen that the two nondimensionalized amplitudes of the diffraction patterns are equal,

$$V_1(\sigma , \tau) = V(\sigma , \tau) \quad ,$$

provided that

$$\begin{cases} A_1 = A_0 \\ \overline{R} = \overline{r} \\ \mathcal{J}_1(u_0\ ,\ v_0) = \mathcal{J}(u_0\ ,\ v_0) \end{cases} ,$$

because the integrands are identical. The first relation means that the amplitudes of the two incident waves must be equal. The second relation may be written as

$$k_1 R = kr$$

or

$$R = \frac{\lambda_1}{\lambda}\, r \quad ,$$

so that the distance scales as the wavelength ratio. The last relation can be written as

$$T_1\left(\frac{kx_0}{k_1}\ ,\ \frac{ky_0}{k_1}\right) = T(x_0\ ,\ y_0) \quad .$$

This implies the spatial correspondence

$$(X_1\ ,\ Y_1) = \left(\frac{kx_0}{k_1}\ ,\ \frac{ky_0}{k_1}\right) = \left(\frac{\lambda_1}{\lambda}\, x_0\ ,\ \frac{\lambda_1}{\lambda}\, y_0\right) \quad ,$$

so that the pattern T_1 is similar to that of T but scaled by the ratio λ_1/λ.

Finally, the equality of the nondimensionalized diffraction amplitudes implies that

$$U_1\left(\frac{k}{k_1}\,\xi\ ,\ \frac{k}{k_1}\,\eta\right) = U(\xi\ ,\ \eta) \quad ,$$

so that the model diffraction pattern is scaled from the original by λ_1/λ.

Another way to view this scaling is that all dimensions in the model system are the same number of wavelengths λ_1 as the corresponding dimensions in the original system in terms of λ_1. It is intuitively satisfying that optical systems should satisfy such a scaling law, but, as indicated earlier, this is not helpful in arriving at a scale model for the field-scale optical pattern recognition system; another scaling law is needed. In particular, it might be helpful if the scaling law allowed one to choose the ratio of the sizes of the real diffraction patterns. That case will be examined next.

III. GENERAL SCALING EXAMPLE

For this example, let α be the scaling factor between the real diffraction patterns of the two systems illustrated in Figure 1; i.e., the pattern amplitudes are related by

$$U_1(\alpha\xi_1\alpha\eta) = U(\xi_1\eta) \quad ,$$

and the spatial correspondence is

$$(\xi_1 \, , \, \eta_1) = (\alpha\xi \, , \, \alpha\eta) \quad .$$

If $\alpha > 1$, the pattern in the model system is larger than that in the original system; and if $\alpha < 1$, the pattern in the model system is smaller than that in the original system.

In terms of the dimensionless variables used in the preceding section, the specified scaling equation can be written as

$$U_1\left(\frac{\alpha\sigma}{k} \, , \, \frac{\alpha\tau}{k}\right) = U\left(\frac{\sigma}{k} \, , \, \frac{\tau}{k}\right) = V(\sigma \, , \, \tau) \quad .$$

But,

$$U_1\left(\frac{\alpha\sigma}{k} \, , \, \frac{\alpha\tau}{k}\right) = U_1\left[\frac{\left(\alpha k_1 \frac{\sigma}{k}\right)}{k_1} \, , \, \frac{\left(\alpha k_1 \frac{\tau}{k}\right)}{k_1}\right] = V_1\left(\frac{\alpha k_1}{k}\sigma \, , \, \frac{\alpha k_1}{k}\tau\right) \quad .$$

Introducing

$$\beta = \frac{\alpha k_1}{k} \quad ,$$

one can write the scaling equation as

$$V_1(\beta\sigma \, , \, \beta\tau) = V(\sigma \, , \, \tau) \quad .$$

From the preceding section it can be seen that

$$V_1(\beta\sigma\ ,\ \beta\tau) = -\frac{A_1 i}{2\pi}\iint \mathcal{J}_1(\bar{u}_0\ ,\ \bar{v}_0)$$

$$\times \frac{e^{i\left[(\bar{u}_0-\beta\sigma)^2+(\bar{v}_0-\beta\tau)^2+\bar{R}^2\right]^{1/2}}}{\left[\bar{u}_0 - \beta\sigma)^2 + (\bar{v}_0 - \beta\tau)^2 + \bar{R}^2\right]^{1/2}}\, d\bar{u}_0 d\bar{v}_0 \quad .$$

Making the obvious substitution

$$\bar{u}_0 = \beta u_0\ ,\ \bar{v}_0 = \beta v_0$$

allows one to write

$$V_1(\beta\sigma\ ,\ \beta\tau) = -\frac{A_1 i\beta^2}{2\pi}\iint \mathcal{J}_1(\beta u_0\ ,\ \beta v_0)$$

$$\times \frac{e^{i\left[\beta^2(u_0-\sigma)^2+\beta^2(v_0-\tau)^2+\bar{R}^2\right]^{1/2}}}{\left[\beta^2(u_0 - \sigma)^2 + \beta^2(v_0 - \tau)^2 + \bar{R}^2\right]^{1/2}}\, du_0 dv_0 \quad ,$$

which is to be compared with the expression for $V(\sigma, \tau)$. From this comparison it must be concluded that for arbitrary β and for arbitrary but physically reasonable functions $\mathcal{J}$ and $\mathcal{J}_1$,

$$V_1(\beta\sigma\ ,\ \beta\tau) \neq V(\sigma\ ,\ \tau) \quad ;$$

i.e., the desired scaling equation is not satisfied.

However, it should be noted that the expressions for V_1 and V are based on the general form of the Fresnel-Kirchhoff integral which applies to a broader class of situations than are normally encountered in optical systems. Therefore, it is reasonable to consider the question of whether the proposed scaling equation can be satisfied for a certain class of optical systems. This question is approached by making certain standard approximations in the Fresnel-Kirchhoff integral.

First, it is observed that in most optical systems the exponential turn in the integrand, for example,

$$e^{ik\left[(x_0-\xi)^2+(y_0-\eta)^2+r^2\right]^{1/2}} \quad ,$$

oscillates over a complete cycle for a one-wavelength change in the distance,

$$S = \left[(x_0 - \xi)^2 + (y_0 - \eta)^2 + r^2\right]^{1/2} \quad .$$

Thus, for even a relatively large change in S, the effect of S in the denominator is practically constant so that it can be replaced by its approximate value r and removed from the integral.

Second, the finite aperture of any real optical system will limit the range of variation of S. This means that S can be replaced in the exponent by an approximation that takes into account the aperture size. Expanding the square root,

$$S = r\left\{1 + \frac{(x_0 - \xi)^2 + (y_0 - \eta)^2}{2r^2} - \frac{\left[(x_0 - \xi)^2 + (y_0 - \eta)^2\right]^2}{8r^4} + \ldots\right\}$$

$$\cong r + \frac{(x_0 - \xi)^2 + (y_0 - \eta)^2}{2r} \quad ,$$

provided that

$$\frac{k\left[(x_0 - \xi)^2 + (y_0 - \eta)^2\right]^2}{8r^3} << 2\pi$$

over the range of variation of the variables. The largest angle that a light ray can make with the optical axis is given by

$$\tan \theta_x = \frac{(x_0 - \xi)\ \max}{r}$$

or

$$\tan \theta_y = \frac{(y_0 - \eta)\ \max}{r} \quad .$$

If one takes $\tan \theta_x = \tan \theta_y$, the preceding inequality becomes

$$\tan^4 \theta << \frac{2\lambda}{r}$$

or

$$\tan \theta << \left(\frac{2\lambda}{r}\right)^{1/4} \quad .$$

For $\lambda = 10.6\ \mu m$ and $r = 10$ km, the righthand side is 6.8×10^{-3}, so that $\theta \ll 23.78$ minutes. For $\lambda = 0.6328\ \mu m$ and $r = 1$ m, the righthand side is 3.4×10^{-2}, so that $\theta \ll 1.92$ degrees.

It has been found from experience that the preceding inequality is more stringent than necessary for most practical calculations. The point is that the target size and aperture size must be small enough, relative to the range, so that all light rays make small angles with the optical axis. Because of this condition, this approximation for S, in which nothing beyond the quadratic terms in the variables is retained, is called the "paraxial approximation."

Upon applying these two approximations, it is found that

$$V(\sigma\ ,\ \tau) = \frac{A_0 i}{2\pi \overline{r}}\, e^{i\overline{r}}\, e^{i/2\overline{r}(\sigma^2+\tau^2)} \times \iint \mathcal{J}(u_0\ ,\ v_0)\, e^{i/2\overline{r}\left(u_0^2+v_0^2\right)}\, e^{i/\overline{r}(u_0\sigma+v_0\tau)}\, du_0 dv_0 \quad ,$$

and that

$$V_1(\beta\sigma\ ,\ \beta\tau) = -\frac{A_1 i\, \beta^2}{2\pi \overline{R}}\, e^{i\overline{R}}\, e^{i\beta^2/2\overline{R}(\sigma^2+\tau^2)} \times \iint \mathcal{J}_1(\beta u_0\ ,\ \beta v_0)\, e^{i\beta^2/2\overline{R}\left(u_0^2+v_0^2\right)}\, e^{-i\beta^2/\overline{R}(u_0\sigma+v_0\tau)}\, du_0 dv_0 \quad .$$

It is now apparent that the scaling equation is satisfied for a broad class of functions $\mathcal{J}(u_0,\ v_0)$ and for β arbitrary provided that

$$\begin{cases} \dfrac{A_1\beta^2}{\overline{R}}\, e^{i\overline{R}} = \dfrac{A_0}{\overline{r}}\, e^{i\overline{r}} \\[2ex] \dfrac{\beta^2}{\overline{R}} = \dfrac{1}{\overline{r}} \\[2ex] \mathcal{J}_1(\beta u_0\ ,\ \beta v_0) = \mathcal{J}(u_0\ ,\ v_0) \quad . \end{cases}$$

It is seen that for $\beta = 1$, the previous wavelength scaling law is obtained.

For arbitrary β, the second of the preceding equations is equivalent to

$$R = \beta^2\left(\frac{\lambda_1}{\lambda}\right) r \quad .$$

The last relation can be written as

$$T_1\left(\beta \frac{k}{k_1} x_0 , \beta \frac{k}{k_1} y_0\right) = T(x_0 , y_0)$$

or

$$T_1(\alpha x_0 , \alpha y_0) = T(x_0 , y_0) \quad ;$$

and the first relation may be written as

$$A_1 = A_0 \, e^{i(1-\beta^2)kr} \quad .$$

Because r, k, and β are fixed numbers, the exponential factor simply represents a fixed-phase relation between the two incident beams. It may be noted that this phase relation will not affect the intensity of the model diffraction pattern.

This example shows that the new scaling relation,

$$U_1(\alpha\xi , \alpha\eta) = U(\xi_1 \eta) \quad ,$$

between a pair of optical systems involving free-space diffraction can be valid as long as the systems satisfy the paraxial approximation. The implication of this result is that by limiting consideration to paraxial systems, an entirely new set of optical-scaling relations can be found.

Before considering these new relations, it is useful to comment further on the present example. For $\alpha = 1$ (i.e., the two diffraction patterns are the same size), one finds that the two objects must be identical; and because $\beta = \lambda/\lambda_1$ in this case, the distances are related by

$$R = \frac{\lambda}{\lambda_1} r \quad .$$

The distance is inversely proportional to the wavelength; whereas, in wavelength scaling, the distance is directly proportional to the wavelength.

The present example also allows scaling of systems with no wavelength change. Taking $\lambda_1 = \lambda$, it is seen that $\beta = \alpha$ and that

$$R = \alpha^2 r \quad .$$

For example, for $\alpha = 2$, the pattern in the model system will be twice the size of the original pattern and the transmission function in the model system will be twice the size of the original function; however, the distance in the model system must be four times the distance in the original system.

The scaling relations that have been derived in this example are valid for a broad class of transmission functions $\mathcal{T}(u_0, v_0)$. In fact, no restrictions outside the paraxial approximation have been placed on $\mathcal{T}$. Because of this generality, this kind of scaling will be defined as "ordinary scaling."

It is important to note, however, that ordinary scaling does not include all possible cases of optical modeling. There can be special transmission functions or special optical systems for which other scaling relations exist. As an example, suppose that

$$T(x_0 , y_0) = \delta(x_0)\delta(y_0)$$

and that

$$T_1(X_0 , Y_0) = \delta(X_0)\delta(Y_0) \quad .$$

Then,

$$\mathcal{T}(u_0 , v_0) = k^{+2} \, \delta(u_0)\delta(v_0)$$

and

$$\mathcal{T}_1(\beta u_0 , \beta v_0) = \left(\frac{\beta}{k_1}\right)^{-2} \delta(u_0)\delta(v_0) \quad ,$$

so that

$$\mathcal{T}_1(\beta u_0 , \beta v_0) \neq \mathcal{T}(u_0 , v_0) \quad .$$

Nevertheless, direct evaluation of the two integrals shows that

$$V_1(\beta\sigma , \beta\tau) = V(\sigma , \tau) \quad ,$$

provided that

$$\frac{A_1 k_1^2}{\overline{R}} e^{i\overline{R}} = \frac{A_0 k^2}{\overline{r}} e^{i\overline{r}} \quad ,$$

and

$$\frac{\beta^2}{\overline{R}} = \frac{1}{\overline{r}} \quad .$$

This kind of modeling (i.e., dependent upon special properties of the objects or of the optical systems) will be called "exceptional scaling" or "singular scaling." Although singular scaling may be of practical importance in certain instances, it is not sufficiently general for the purposes of the optical pattern recognition work. This report, therefore, is concerned primarily with ordinary scaling.

IV. DEFINITIONS AND RULES OF ORDINARY SCALING

Ordinary scaling applies to optical systems for which the paraxial approximation is valid and in which the light is totally coherent. Ordinary scaling is independent of special structural properties of the object and of special characteristics of the optical system, such as the formation of an image or a Fraunhofer diffraction pattern in the output plane. Ordinary scaling requires a geometrical similarity, at the output planes, between the total diffraction effects of the two optical systems, the original system, and the model system.

Because ordinary scaling is based on diffraction phenomena, it is applicable to any optical pattern recognition system, optical data processing system, or imaging system using coherent light. It should also be applicable to holography, although no attempt has been made to examine this question.

The principles of ordinary optical scaling are sufficiently general that they can be applied to systems containing any kind of optical element including electrooptic or acoustooptic devices and nonlinear elements, as well as ordinary passive elements. These principles are also expected to be applicable to propagation through turbulent media and through scattering media. However, general investigations have not been undertaken. It is sufficient for present purposes to consider optical systems consisting only of spherical lenses and apertures.

The primary scaling relation is the geometrical similarity between diffraction patterns. Denoting the model system by a subscript, on the functions as before, this similarity can be expressed as

$$\text{I.} \quad U_1\left(\frac{\beta k}{k_1}\xi\ ,\ \frac{\beta k}{k_1}\eta\right) = U(\xi\ ,\ \eta) \quad .$$

The functions U, U_1 are complex amplitudes, and this relation says that the amplitudes are geometrically similar in all respects, including the phase. The phase can be important in some applications, holography being an example, but in the final output plane of most systems the phase is unimportant because the intensity, or absolute square of the amplitude, is measured. For these systems, condition I can be replaced by

$$\text{II.} \quad U_1\left(\frac{\beta k}{k_1}\xi\ ,\ \frac{\beta k}{k_1}\eta\right) = K\,U(\xi\ ,\ \eta) \quad ,$$

where K is an arbitrary phase function involving the output coordinates (ξ, η) as well as geometrical constants of the primary optical system. Condition II is less restrictive than condition I. Condition II has the particular virtue that it removes any requirement to control the phase of the input light beam on the model system.* Because it is a pure phase function, K satisfies the relation

$$|K|^2 = 1 \quad .$$

In terms of the nondimensionalized amplitudes, the two preceding conditions are

$$\text{I}'. \quad V_1(\beta\sigma \ , \ \beta\tau) = V(\sigma \ , \ \tau)$$

and

$$\text{II}'. \quad V_1(\beta\sigma \ , \ \beta\tau) = K\,V(\sigma \ , \ \tau) \quad .$$

In the remainder of this work, only conditions II and II' will be used.

One more remark may be made regarding the basic scaling relations. As written, the relations require that the two systems have the same power. This requirement can be convenient for testing detectors on a model, and it will be carried through the calculations. The equal power requirement does not affect the other parameters of the model. If power is not a consideration, one can simply let $|K|^2$ be any convenient constant.

The equations for the ordinary scaling of one optical system from another can be developed in a straightforward fashion by following the type of procedure that was used with the example in the second section. This procedure is formalized in the following seven rules:

Rule 1 — The total diffraction effect of each of the optical systems is written as a multiple integral by successive application of the Fresnel-Kirchhoff diffraction integral.

Rule 2 — The paraxial approximation is applied.

*For systems in which the intensity is measured in the output plane, the scaling relation could be taken as

$$\left| U_1\left(\frac{\beta k}{k_1}\,\xi \ , \ \frac{\beta k}{k_1}\,\eta\right)\right|^2 = |U(\xi \ , \ \eta)|^2 \quad .$$

Rule 3 — No special properties of the optical system or the object are to be introduced into the integrals, and no integrations are to be carried out, no matter how simple.

Rule 4 — The integrals are nondimensionalized.

Rule 5 — Scaling factors are introduced into the integral for the model system. There will be one scaling factor for each internal plane or independent coordinate, and one scaling factor for the output plane.

Rule 6 — Factors independent of the variables of integration are moved outside each multiple integral and lumped together as a phase-amplitude coefficient.

Rule 7 — The scaling condition II' is satisfied by requiring equality of corresponding independent factors in the integrands for the two systems.

Rule 3 helps to guarantee that no singular properties of the object or a particular optical system, e.g., one having an image plane in the output plane, are introduced into the scaling equations. The prohibition against carrying out any integrations guarantees that the maximum number of scaling factors remain in the scaling equations. The fact that a particular kind of optical element (lens or prism, for example) is used at a given location must be introduced into the multiple integral for the diffraction effects; however, numerical values and special geometrical relationships should be excluded.

These rules generate scaling equations for classes of optical systems; i.e., a set of equations for one-lens systems, a set for two-lens systems, etc. Numerical values and special relations for a particular system in a class can then be substituted into the appropriate equations.

It is convenient for present purposes to treat the illuminating light beam in a specialized way independent of Rule 3. This avoids some complexities and specializes the results to systems similar to the suggested design of the tactical optical pattern recognition system. A discussion of the illuminating beam will be presented in the next section. Then, the preceding seven rules will be applied, in succession, to systems with increasing numbers of elements. The discussion will end with a set of equations that are appropriate for the tactical optical pattern recognition system.

V. TARGET ILLUMINATION

The first two examples in this report were based on plane-wave illumination of the target. This was chosen to illustrate the two scaling principles in the simplest manner. Any tactical optical pattern recognition system using a laser source must use a diverging spherical wave to illuminate the target. The target illumination and diffraction to the first optical surface is illustrated in Figure 2. For computational purposes, the targets are represented as transparencies with the light sources behind them. Reflection and transmission are entirely equivalent optically.

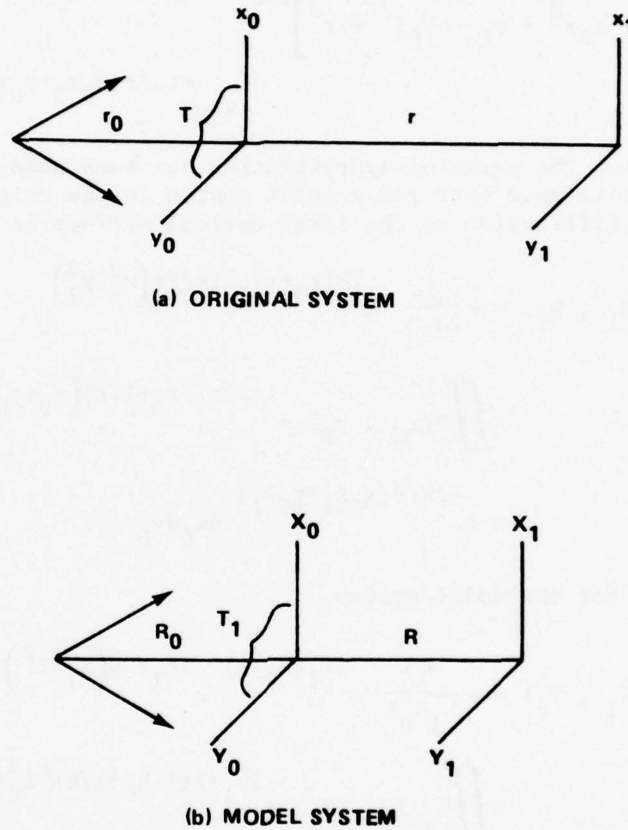


Figure 2. Target illumination and diffraction to first optical surface.

In the original system, the contribution from the spherical wave illumination is

$$\frac{e^{ik\left[x_0^2+y_0^2+r_0^2\right]^{1/2}}}{\left[x_0^2+y_0^2+r_0^2\right]^{1/2}} \cong \frac{e^{ikr_0}}{r_0}\; e^{ik/2r_0\left(x_0^2+y_0^2\right)} ,$$

where the paraxial approximation has been used. A similar expression is true for the model system. The propagation factor from the object to the x_1, y_1 plane is

$$\frac{e^{ik\left[(x_1-x_0)^2+(y_1-y_0)^2+r^2\right]^{1/2}}}{\left[(x_1-x_0)^2+y_1-y_0)^2+r^2\right]^{1/2}} \cong \frac{e^{ikr}}{r}\, e^{i\,k/2r\left(x_1^2+y_1^2\right)}\, e^{ik/2r\left(x_0^2+y_0^2\right)}$$

$$\times\, e^{-ik/r(x_0x_1+y_0y_1)} \quad ,$$

where, again, the paraxial approximation has been used. From these results, it is seen that for a point source in the original system the free-space diffraction to the first optical surface is given by

$$U^{(1)}(x_1\ ,\ y_1) = -\frac{Ai}{\lambda r_0 r}\, e^{ik(r_0+r)}\, e^{ik/2r\left(x_1^2+y_1^2\right)}$$

$$\times\iint T(x_0\ ,\ y_0)\, e^{ik/2(1/r_0+1/r)\left(x_0^2+y_0^2\right)}$$

$$\times\, e^{-ik/r(x_0x_1+y_0y_1)}\, dx_0dy_0 \quad .$$

Similarly, for the model system

$$U_1^{(1)}(X_1\ ,\ Y_1) = -\frac{A_1 i}{\lambda_1 R_0 R}\, e^{ik_1(R_0+R)}\, e^{ik_1/2R\left(X_1^2+Y_1^2\right)}\ ,$$

$$\times\iint T_1(X_1\ ,\ Y_1)\, e^{ik_1/2(1/R_0+1/R)\left(X_0^2+Y_0^2\right)}$$

$$\times\, e^{-ik_1/R(X_0X_1+Y_0Y_1)}\, dX_0dY_0 \quad .$$

These are the starting expressions for the subsequent development of scaling equations.

VI. SCALING EQUATIONS FOR A ONE-LENS SYSTEM

The two systems are illustrated in Figure 3. The lenses are designated by ℓ and $\mathcal{L}$, respectively, and have focal lengths f and F as

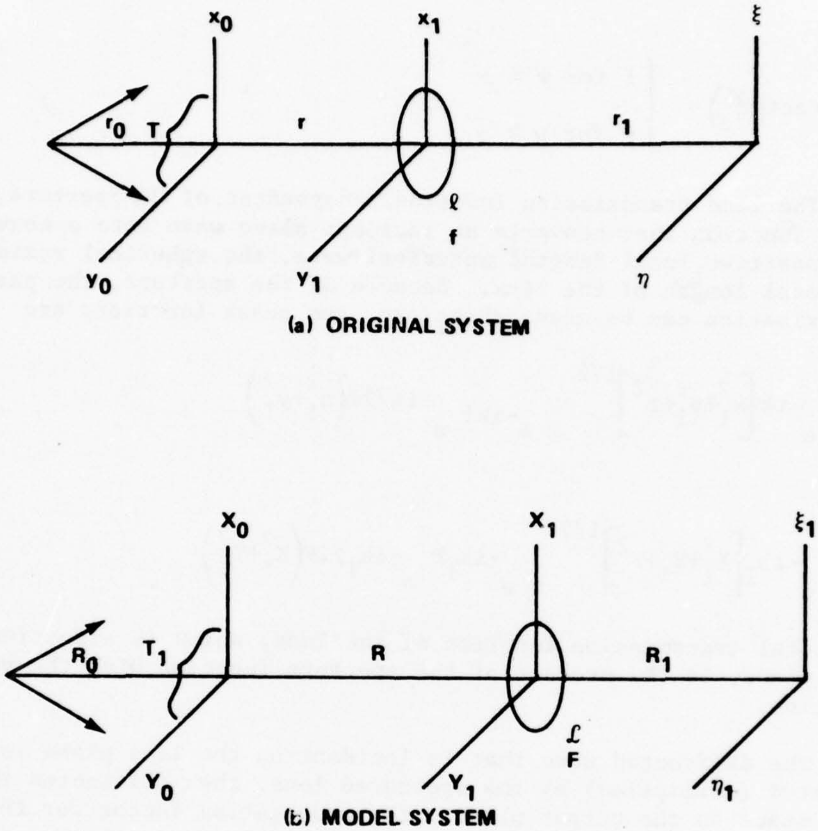


Figure 3. One-lens system.

indicated. The lenses are thin spherical lenses with circular apertures. The apertures are represented by the transmission functions

$$D(x_1\ ,\ y_1\ ;\ \rho) = \mathrm{rect}\left[\frac{\left(x_1^2 + y_1^2\right)^{1/2}}{2\rho}\right]$$

and

$$D_1(X_1\ ,\ Y_1\ ;\ \rho) = \mathrm{rect}\left[\frac{\left(X_1^2 + Y_1^2\right)^{1/2}}{2\rho_1}\right]\quad ,$$

where

$$\mathrm{rect}\left(\frac{w}{2\rho}\right) = \begin{cases} 1 \text{ for } w \le \rho \\ 0 \text{ for } w > \rho \end{cases} .$$

The lens transmission function, independent of the aperture, is a pure-phase function that converts an incident plane wave into a converging (for positive focal length) spherical wave, the spherical radius being the focal length of the lens. Because of the aperture, the paraxial approximation can be used; therefore, the phase functions are

$$e^{-ik\left[x_1^2+y_1^2+f^2\right]^{1/2}} \cong e^{-ikf} e^{-ik/2f\left(x_1^2+y_1^2\right)}$$

and

$$e^{-ik_1\left[X_1^2+Y_1^2+F^2\right]^{1/2}} \cong e^{-ik_1F} e^{-ik_1/2F\left(X_1^2+Y_1^2\right)} .$$

The total transmission function of the lens, which is effective at the lens plane, is the product of the aperture function with the phase function.

The diffracted wave that is incident on the lens plane is first affected (multiplied) by the apertured lens, then diffracted through free space to the output plane. The propagation factor for this last diffraction in the original system can be written in the paraxial approximation as

$$\frac{e^{ik\left[(\xi-x_1)^2+(\eta-y_1)^2+r_1^2\right]^{1/2}}}{\left[(\xi - x_1)^2 + (\eta - y_1)^2 + r_1^2\right]^{1/2}} \cong \frac{e^{ikr_1}}{r_1} e^{ik/2r_1(\xi^2+\eta^2)} e^{ik/2r_1\left(x_1^2+y_1^2\right)} \times e^{-ik/r_1(x_1\xi+y_1\eta)} ,$$

with a similar expression for the model system. Putting these results together with the previous results on target illumination, it is found that

$$U(\xi , \eta) = \left(-\frac{i}{\lambda}\right)^2 A_0 \frac{e^{ik(r_0+r+r_1-f)}}{r_0 r_1 r} e^{ik/2r_1(\xi^2+\eta^2)}$$

$$\times \iiiint T(x_0 , y_0)\, e^{ik/2(1/r_0+1/r)\left(x_0^2+y_0^2\right)}$$

$$\times D(x_1 , y_1 , \rho)\, e^{ik/2(1/r+1/r_1-1/f)\left(x_1^2+y_1^2\right)}$$

$$\times e^{-ik[x_1(x_0/r+\xi/r_1)+y_1(y_0/r+\eta/r_1)]} dx_0 dy_0 dx_1 dy_1 \quad ,$$

and

$$U_1(\xi_1 , \eta_1) = \left(-\frac{i}{\lambda}\right)^2 A_1 \frac{e^{ik_1(R_0+R+R_1-F)}}{R_0 R_1 R} e^{ik_1/2R_1\left(\xi_1^2+\eta_1^2\right)}$$

$$\times \iiiint T_1(X_0 , Y_0)\, e^{ik_1/2(1/R_0+1/R)\left(X_0^2+Y_0^2\right)}$$

$$\times D_1(X_1 , Y_1 ; \rho_1)\, e^{ik_1/2(1/R+1/R_1-1/F)\left(X_1^2+Y_1^2\right)}$$

$$\times e^{-ik_1[X_1(X_0/R+\xi_1/R_1)+Y_1(Y_0/R+\eta_1/R_1)]} dX_0 dY_0 dX_1 dY_1 \quad .$$

To nondimensionalize the diffraction equation for the original system, the change of variables is introduced:

$$\begin{cases} u_k = kx_i \ , \ v_i = ky_i \\ \bar{r}_i = kr_i \ , \ \bar{f} = kf \\ \sigma = k\xi \ , \ \tau = k\eta \end{cases} ,$$

and also the functions

$$\begin{cases} V(\sigma\,,\tau) = U\left(\frac{\sigma}{k}\,,\,\frac{\tau}{k}\right) = U(\xi\,,\eta) \\ \mathcal{T}(u_0\,,v_0) = T\left(\frac{u_0}{k}\,,\,\frac{v_0}{k}\right) = T(x_0\,,y_0) \\ D(u_1\,,v_1\,;k\rho) = D\left(\frac{u_1}{k}\,,\,\frac{v_1}{k}\,;\,\rho\right) = D(x_1\,,y_1\,;\rho) \end{cases} \quad .$$

Then, one can write

$$V(\sigma\,,\tau) = \frac{(-i)^2}{2\pi\lambda} A_0 \frac{e^{i(\overline{r}_0+\overline{r}+\overline{r}_1-\overline{f})}}{\overline{r}_0\overline{r}\,\overline{r}_1} e^{i/2\overline{r}_1(\sigma^2+\tau^2)}$$

$$\times \iint \mathcal{T}(u_0\,,v_0)\, e^{i/2(1/\overline{r}_0+1/\overline{r})\left(u_0^2+v_0^2\right)}$$

$$\times\, I(u_0\,,v_0\,;\sigma\,,\tau)\, du_0 dv_0 \quad ,$$

where

$$I(u_0\,,v_0\,;\sigma\,,\tau) = \iint D(u_1\,,v_1\,;k\rho)$$

$$\times\, e^{i/2(1/\overline{r}+1/\overline{r}_1-1/\overline{f})\left(u_1^2+v_1^2\right)}$$

$$\times\, e^{-i[u_1(u_0/\overline{r}+\sigma/\overline{r}_1)+v_1(v_0/\overline{r}+\tau/\overline{r}_1)]}\, du_1 dv_1 \quad .$$

It is convenient, for the application of the last rule, to separate the multiple integral in this fashion.

For the model system, the first change of variables is

$$\begin{cases} \overline{u}_i = k_1 X_i\,,\overline{v}_i = k_1 Y_i \\ \overline{R}_i = k_1 R_i\,,\overline{F} = k_1 F \\ \sigma_1 = k_1\xi_1\,,\tau_1 = k_1\eta_1 \end{cases} \quad ,$$

This change is accompanied by the functional substitutions

$$\begin{cases} V_1(\sigma_1, \tau_1) = U_1\left(\frac{\sigma_1}{k_1}, \frac{\tau_1}{k_1}\right) = U_1(\xi_1, \eta_1) \\ \mathcal{T}_1(\overline{u}_0, \overline{v}_0) = T_1\left(\frac{\overline{u}_0}{k_1}, \frac{\overline{v}_0}{k_1}\right) = T_1(X_0, Y_0) \\ D_1(\overline{u}_1, \overline{v}_1; k_1\rho_1) = D_1\left(\frac{\overline{u}_1}{k_1}, \frac{\overline{v}_1}{k_1}; \rho_1\right) = D_1(X_1, Y_1; \rho_1) \end{cases},$$

so that one obtains

$$V_1(\sigma_1, \tau_1) = \frac{(-i)^2}{2\pi\lambda_1} A_1 \frac{e^{i(\overline{R}_0+\overline{R}+\overline{R}_1-\overline{F})}}{\overline{R}_0\overline{R}\,\overline{R}_1} e^{i/2R_1\left(\sigma_1^2+\tau_1^2\right)}$$
$$\times\iint \mathcal{T}_1(\overline{u}_0, \overline{v}_0)\, e^{i/2(1/\overline{R}_0+1/\overline{R})\left(\overline{u}_0^2+\overline{v}_0^2\right)}$$
$$\times\, I_1(\overline{u}_0, \overline{v}_0; \sigma_1, \tau_1)\, d\overline{u}_0 d\overline{v}_0 ,$$

where

$$I_1(\overline{u}_0, \overline{v}_0; \sigma_1, \tau_1) = \iint D_1(\overline{u}_1, \overline{v}_1; k_1\rho)$$
$$\times\, e^{i/2(1/\overline{R}+1/\overline{R}_1-1/\overline{F})\left(\overline{u}_1^2+\overline{v}_1^2\right)}$$
$$\times\, e^{-i[\overline{u}_1(\overline{u}_0/\overline{R}+\sigma_1/\overline{R}_1)+\overline{v}_1(\overline{v}_0/\overline{R}+\tau_1/\overline{R}_1)]}\, d\overline{u}_1 d\overline{v}_1 .$$

At this stage, the scaling factors are introduced through the change of variables

$$\begin{cases} \overline{u}_0 = \alpha u_0 , \overline{v}_0 = \alpha v_0 \\ \overline{u}_1 = \gamma u_1 , \overline{v}_1 = \gamma v_1 \\ \sigma_1 = \beta\sigma , \tau_1 = \beta\tau \end{cases}.$$

This gives

$$V_1(\beta\sigma, \beta\tau) = \frac{(-i\alpha\gamma)^2}{2\pi\lambda_1} A_1 \frac{e^{i(\overline{R}_0+\overline{R}+\overline{R}_1-\overline{F})}}{\overline{R}_0\overline{R}\,\overline{R}_1} e^{i\beta^2/2\overline{R}_1(\sigma^2+\tau^2)}$$

$$\times\iint \mathcal{J}_1(\alpha u_0, \alpha v_0)\; e^{i\alpha^2/2(1/\overline{R}_0+1/\overline{R})\left(u_0^2+v_0^2\right)}$$

$$\times\; I_1(\alpha u_0, \alpha v_0; \beta\sigma, \beta\tau)\; du_0 dv_0 \quad ,$$

and

$$I_1(\alpha u_0, \alpha v_0; \beta\sigma, \beta\tau) = \iint D_1(\gamma u_1, \gamma v_1; k_1\rho_1)$$

$$\times\; e^{i\gamma^2/2(1/\overline{R}+1/\overline{R}_1-1/\overline{F})\left(u_1^2+v_1^2\right)}$$

$$\times\; e^{-i\gamma\left[u_1(\alpha u_0/\overline{R}+\beta\sigma/\overline{R}_1)+v_1(\alpha v_0/\overline{R}+\beta\tau/\overline{R}_1)\right]}\, du_1 dv_1 \quad .$$

Applying Rule 7, first set $I_1 = I$, then apply the rule directly to the two integrands. These first steps yield the following relations:

$$\begin{cases} \dfrac{\beta\gamma}{\overline{R}_1} = \dfrac{1}{\overline{r}_1} \\[2ex] \dfrac{\alpha\gamma}{\overline{R}} = \dfrac{1}{\overline{r}} \\[2ex] \gamma^2\left(\dfrac{1}{\overline{R}} + \dfrac{1}{\overline{R}_1} - \dfrac{1}{\overline{F}}\right) = \left(\dfrac{1}{\overline{r}} + \dfrac{1}{\overline{r}_1} - \dfrac{1}{\overline{f}}\right) \\[2ex] D_1(\gamma u_1, \gamma v_1; k_1\rho_1) = D(u_1, v_1; k\rho) \quad . \end{cases}$$

Next, Rule 7 is applied to the remaining factors in the integrals for V and V_1. This gives the relations

$$\begin{cases} \alpha^2\left(\dfrac{1}{\overline{R}_0} + \dfrac{1}{\overline{R}}\right) = \left(\dfrac{1}{\overline{r}_0} + \dfrac{1}{\overline{r}}\right) \\[2ex] \mathcal{J}_1(\alpha u_0, \alpha v_0) = \mathcal{J}(u_0, v_0) \quad . \end{cases}$$

Finally, using the parameter K to remove the pure phase factors, Rule 7 applied to the coefficients of the integrals gives

$$\frac{\alpha^2\gamma^2}{\lambda_1} A_1 \frac{1}{R_0 R R_1} = \frac{1}{\lambda} A_0 \frac{1}{\bar{r}_0 \bar{r} \bar{r}_1} \quad .$$

These seven equations, together with II', constitute the scaling equations, but they are more conveniently expressed in terms of the physical coordinates and functions. Collecting the distance relations, one finds that

$$\begin{cases} \alpha^2\left(\dfrac{1}{R_0} + \dfrac{1}{R}\right) = \dfrac{\lambda}{\lambda_1}\left(\dfrac{1}{r_0} + \dfrac{1}{r}\right) \\[2ex] \dfrac{\alpha\gamma}{R} = \dfrac{\lambda}{\lambda_1} \dfrac{1}{r} \\[2ex] \dfrac{\beta\gamma}{R_1} = \dfrac{\lambda}{\lambda_1} \dfrac{1}{r_1} \\[2ex] \gamma^2\left(\dfrac{1}{R} + \dfrac{1}{R_1} - \dfrac{1}{F}\right) = \dfrac{\lambda}{\lambda_1}\left(\dfrac{1}{r} + \dfrac{1}{r_1} - \dfrac{1}{f}\right) \end{cases} \quad .$$

If the scaling factors α, β, γ are considered completely free variables, the first three equations uniquely define the model distances R_0, R, R_1. The fourth equation then uniquely defines F. A little algebraic manipulation shows that the last equation can be written as

$$\frac{1}{F} = \frac{\lambda}{\gamma^2\lambda_1}\left[\frac{1}{f} - \frac{(\alpha - \gamma)}{\alpha}\frac{1}{r} - \frac{(\beta - \gamma)}{\beta}\frac{1}{r_1}\right] \quad .$$

The equation relating the aperture functions is also a single distance equation because of the special form of the aperture functions. It is found that

$$D_1(\gamma u_1 , \gamma v_1 ; k_1\rho_1) = \text{rect}\left[\frac{\left(u_1^2 + v_1^2\right)^{1/2}}{2\left(\dfrac{k_1\rho_1}{\gamma}\right)}\right] \quad ,$$

and that

$$D(u_1 , v_1 ; k\rho) = \text{rect}\left[\frac{\left(u_1^2 + v_1^2\right)^{1/2}}{2k\rho}\right] \quad ;$$

therefore, the aperture scaling equation is

$$\rho_1 = \frac{\gamma\lambda_1}{\lambda}\rho \quad .$$

The functional scaling relations are

$$\begin{cases} U_1\left(\frac{\beta\lambda_1}{\lambda}\xi \, , \, \frac{\beta\lambda_1}{\lambda}\eta\right) = K\, U(\xi \, , \eta) \\ T_1\left(\frac{\alpha\lambda_1}{\lambda}x_0 \, , \, \frac{\alpha\lambda_1}{\lambda}y_0\right) = T(x_0 \, , y_0) \end{cases} \quad .$$

It is important to note that now, with a lens in the system, the scale factors on the object and the diffraction pattern are not the same.

Finally, the relation between the amplitudes of the input illuminating beams is

$$A_1 = \frac{\lambda_1}{\lambda}\alpha\beta\gamma\, A_0 \frac{r}{[\gamma r - (\alpha - \gamma)r_0]} \quad .$$

It is this instructive to consider some examples. First, suppose that it is desired to have the two diffraction patterns the same size, second, that it is desired to reduce the target in the model by a factor of 100, and third, that for reasons of symmetry

$$\frac{R_0}{R} = \frac{r_0}{r} \quad .$$

It can be shown that this equation implies that

$$\gamma = \alpha \quad .$$

The first two conditions previously given are equivalent to

$$\frac{\beta\lambda_1}{\lambda} = 1$$

and

$$\frac{\alpha\lambda_1}{\lambda} = 10^{-2} \quad .$$

Using these relations in the first three distance equations, one obtains

$$R_0 = 10^{-4} \frac{\lambda}{\lambda_1} r_0$$

$$R = 10^{-4} \frac{\lambda}{\lambda_1} r$$

$$R_1 = 10^{-2} \frac{\lambda}{\lambda_1} r_1 \quad .$$

The equation for the model focal length becomes

$$\frac{1}{F} = \frac{\lambda_1}{\lambda} 10^4 \left[\frac{1}{f} - (1 - 10^{-2}) \frac{1}{r_1} \right] \quad .$$

To further specialize the system, suppose that

$$r_1 = f \quad ,$$

so that

$$F = 10^{-2} \frac{\lambda}{\lambda_1} f = 10^{-2} \frac{\lambda}{\lambda_1} r_1 = R_1 \quad .$$

The aperture in the model system is given by

$$\rho_1 = 10^{-2} \rho \quad ,$$

and the input beam amplitude by

$$A_1 = 10^{-2} \frac{\lambda}{\lambda_1} A_0 \quad .$$

Suppose that the wavelength is not changed ($\lambda_1 = \lambda$), that the original system consists of a 6-m target 10 km away, that $r_0 = r$, $f = 1$ m, and $\rho = 12.5$ cm. The model system would then consist of a 6-cm target 1 m away, with $R_0 = R = 1$ m, $F = 1$ cm, and $\rho_1 = 1.25$ mm. The distance R_1 would be 1 cm.

The point of this example is to show directly that this new system of optical scaling can, in fact, be used to reduce a field-scale system to a manageable laboratory size. It is important to note that all quantities do not scale in the same way. This is a consequence not only of the fact that there are three scale factors, but also of the fact that they enter the distance relations in different nonlinear ways.

As another example, an interesting special situation occurs with the scaling parameters $\gamma = \alpha$, $\beta\lambda_1/\lambda = 1$, $\alpha\lambda_1/\lambda = 10^{-2}$ if, in the original system,

$$r_1 = 0.99\ f \quad .$$

Then, the equation for the focal length in the model system gives

$$\frac{1}{F} = 0 \quad ,$$

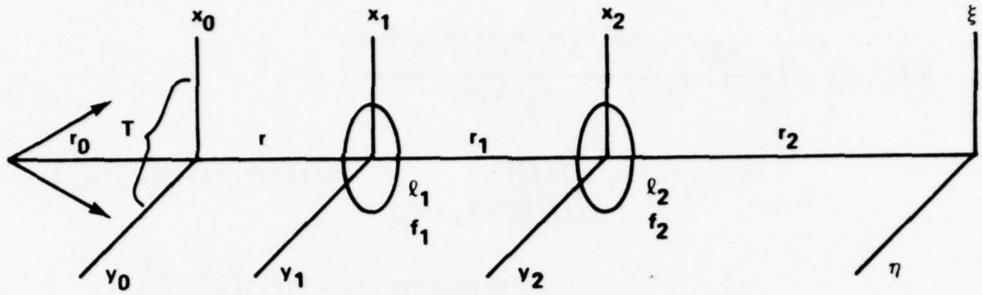
which implies an infinitely long focal length, or no lens at all. Thus, a "particular" system involving a lens and aperture can be modeled to yield the same diffraction pattern with an aperture only. This is a result in ordinary scaling particularized to a very special system. The correctness of the result can be verified directly by substituting the model system parameters into the expression for $U_1(\xi_1, \eta_1)$, and the value of $f = r_1/0.99$ into the expression for $U(\xi_1 \eta)$. The author knows of no other simple and direct way to find this model.

A more general result related to this example can be stated. For $R_0/R = r_0/r$, $\beta\lambda_1/\lambda = 1$, and any $r_1 < f$ a scaling factor α can be found with $\alpha\lambda_1/\lambda < 1$, such that $F = 0$. That is, any single-lens system with the receiving plane inside the focal plane of the lens can be modeled by a lensless system containing an aperture and yielding a diffraction pattern identical to that from the original system. This result is physically reasonable because in either system a Fresnel diffraction pattern is obtained, and it is just a matter of locating corresponding positions along the optical axis. But, again, there is no other simple way of locating these positions.

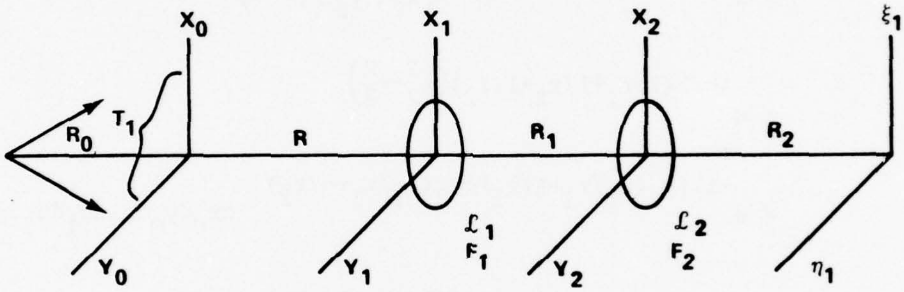
These examples illustrate the power and versatility of this new system of optical scaling.

VII. SCALING EQUATIONS FOR A TWO-LENS SYSTEM

The two systems are illustrated in Figure 4. The lenses are again designated by ℓ_i and $\mathcal{L}_i$ and have focal lengths f_i and F_i. The aperture transmission functions are denoted by $D^{(i)}(x_i, y_i; \rho^{(i)})$ for



(a) ORIGINAL SYSTEM



(b) MODEL SYSTEM

Figure 4. Two-lens system.

the original system and by $D_1^{(i)}\left(X_i, Y_i; \rho_1^{(i)}\right)$ for the model system. It will be assumed later that these are rect-functions, as before. The phase functions for the lenses have the same mathematical structure as before.

The physical change in going from a one-lens to a two-lens system can be thought of as putting the output of the one-lens system into the lens at the x_2, y_2-plane (or the X_2, Y_2-plane), then allowing free-space diffraction to the new output plane. The transition for the mathematical expressions follows this prescription exactly, with the added proviso that each diffraction integral has a factor $(-i/\lambda)$ associated with it.

Thus, using the one-lens expressions, changing (ξ, η) to (x_2, y_2) (and correspondingly for the model system) and introducing the effect of the second lens and the final distance, it is easy to find that

$$U(\xi , \eta) = \left(\frac{-i}{\lambda}\right)^3 A_0 \frac{e^{ik(r_0+r+r_1+r_2-f_1-f_2)}}{r_0 r r_1 r_2}$$

$$\times e^{ik/2r_2(\xi^2+\eta^2)} \iiiint\!\!\iint T(x_0 , y_0)\, e^{ik/2(1/r_0+1/r)\left(x_0^2+y_0^2\right)}$$

$$\times D^{(1)}\left(x_1 , y_1 ; \rho^{(1)}\right) e^{ik/2(1/r+1/r_1-1/f_1)\left(x_1^2+y_1^2\right)}$$

$$\times e^{-ik/r(x_1x_0+y_1y_0)}\, D^{(2)}\left(x_2 , y_2 ; \rho^{(2)}\right)$$

$$\times e^{ik/2(1/r_1+1/r_2-1/f_2)\left(x_2^2+y_2^2\right)}$$

$$\times e^{-ik[x_2(x_1/r_1+\xi/r_2)+y_2(y_1/r_1+\eta/r_2)]}\, dx_0dy_0dx_1dy_1dx_2dy_2 \quad ,$$

and

$$U_1(\xi_1 , \eta_1) = \left(\frac{-i}{\lambda_1}\right)^3 A_1 \frac{e^{ik_1(R_0+R+R_1+R_2-F_1-F_2)}}{R_0 R R_1 R_2}$$

$$\times e^{ik_1/2R_2\left(\xi_1^2+\eta_1^2\right)} \iiiint\!\!\iint T_1(X_0 , Y_0)\, e^{ik_1/2(1/R_0+1/R)\left(X_0^2+Y_0^2\right)}$$

$$\times D_1^{(1)}\left(X_1 , Y_1 ; \rho_1^{(1)}\right) e^{ik_1/2(1/R+1/R_1-1/F_1)\left(X_1^2+Y_1^2\right)}$$

$$\times e^{-ik_1/R(X_1X_0+Y_1Y_0)}\, D_1^{(2)}\left(X_2 , Y_2 ; \rho_1^{(2)}\right)$$

$$\times e^{-ik_1[X_2(X_1/R_1+\xi_1/R_2)+Y_2(Y_1/R_1+\eta_1/R_2)]}\, dX_0dY_0dX_1dY_1dX_2dY_2 \,.$$

For the original system, the following transformations are made:

$u_i = kx_i \, , \, v_i = ky_i$

$\overline{r}_i = kr_i \, , \, \overline{f}_i = kf_i$

$$\sigma = k\xi \;,\; \tau = k\eta$$

$$V(\sigma\,,\tau) = U\left(\frac{\sigma}{k}\,,\frac{\tau}{k}\right) = U(\xi_1\eta)$$

$$\mathcal{T}(u_0\,,v_0) = T\left(\frac{u_0}{k}\;,\;\frac{v_0}{k}\right) = T(x_0\,,y_0)$$

$$D^{(i)}\left(u_i\,,v_i\,;\rho^{(i)}\right) = D^{(i)}\left(x_i\,,y_i\,;\rho^{(i)}\right) \quad .$$

The corresponding transformations for the model system are

$$\overline{u}_i = k_1X_i\,,\overline{v}_i = k_1Y_i$$

$$\overline{R}_i = k_1R_i\,,\overline{F}_i = k_1F_i$$

$$\sigma_1 = k_1\xi_1\,,\tau_1 = k_1\eta_1$$

$$V_1(\sigma_1\,,\tau_1) = U_1\left(\frac{\sigma_1}{k_1}\;,\;\frac{\tau_1}{k_1}\right) = U_1(\xi_1\,,\eta_1)$$

$$\mathcal{T}_1(\overline{u}_0\,,\overline{v}_0) = T_1\left(\frac{\overline{u}_0}{k_1}\;,\;\frac{\overline{v}_0}{k_1}\right) = T_1(X_0\,,Y_0)$$

$$D_1^{(i)}\left(\overline{u}_i\,,\overline{v}_i\,;\rho_1^{(i)}\right) = D_1^{(i)}\left(\frac{\overline{u}_i}{k_1}\;,\;\frac{\overline{v}_i}{k_1}\;;\;\rho_1^{(i)}\right) = D_1^{(i)}\left(X_i\,,Y_i\,;\rho_1^{(i)}\right) \quad .$$

Using these,one finds that

$$V(\sigma\,,\tau) = \frac{(-i)^3}{(2\pi)^2\lambda}\,A_0\,\frac{e^{i(\overline{r}_0+\overline{r}+\overline{r}_1+\overline{r}_2-\overline{f}_1-\overline{f}_2)}}{\overline{r}_0\overline{r}\,\overline{r}_1\overline{r}_2}$$

$$\times\, e^{i/2\overline{r}_2\left(\sigma^2+\tau^2\right)}\iint \mathcal{T}(u_0\,,v_0)\,e^{i/2(1/\overline{r}_0+1/\overline{r})\left(u_0^2+v_0^2\right)}$$

$$\times\, I(u_0\,,v_0\,;\sigma\,,\tau)\;du_0dv_0 \quad ,$$

where

$$I(u_0\,,\,v_0\,;\,\sigma\,,\,\tau) = \iint D^{(1)}\left(u_1\,,\,v_1\,;\,\rho^{(1)}\right)\, e^{i/2(1/\overline{r}+1/\overline{r}_1-1/\overline{f}_1)\left(u_1^2+v_1^2\right)}$$

$$\times\, e^{-i/\overline{r}(u_1u_0+v_1v_0)}\; J(u_1\,,\,v_1\,;\,\sigma\,,\,\tau)\; du_1dv_1 \quad ,$$

and

$$J(u_1\,,\,v_1\,;\,\sigma\,,\,\tau) = \iint D^{(2)}\left(u_2\,,\,v_2\,;\,\rho^{(2)}\right)\, e^{i/2(1/\overline{r}_1+1/\overline{r}_2-1/\overline{f}_2)\left(u_2^2+v_2^2\right)}$$

$$\times\, e^{-i[u_2(u_1/\overline{r}_1+\sigma/\overline{r}_2)+v_2(v_1/\overline{r}_1+\tau/\overline{r}_2)]}\; du_2dv_2 \quad ;$$

and that

$$V(\sigma_1\,,\,\tau_1) = \frac{(-i)^3}{(2\pi)^2\lambda_1}\, A_1\, \frac{e^{i(\overline{R}_0+\overline{R}\pm\overline{R}_1+\overline{R}_2-\overline{F}_1-\overline{F}_2)}}{\overline{R}\;\overline{R}\,\overline{R}_1\overline{R}_2}$$

$$\times\, e^{i/2\overline{R}_2\left(\sigma_1^2+\tau_1^2\right)} \iint \mathcal{J}_1(\overline{u}_0\,,\,\overline{v}_0)\, e^{i/2(1/\overline{R}_0+1/\overline{R})\left(\overline{u}_0^2+\overline{v}_0^2\right)}$$

$$\times\, I_1(\overline{u}_0\,,\,\overline{v}_0\,;\,\sigma_1\,,\,\tau_1)\; d\overline{u}_0\,,\,d\overline{v}_0 \quad ,$$

where

$$I_1(\overline{u}_0\,,\,\overline{v}_0\,;\,\sigma_1\,,\,\tau_1) = \iint D_1^{(1)}\left(\overline{u}_1\,,\,\overline{v}_1\,;\,\rho_1^{(1)}\right)\, e^{i/2(1/\overline{R}+1/\overline{R}_1-1/\overline{F}_1)\left(\overline{u}_1^2+\overline{v}_1^2\right)}$$

$$\times\, e^{-i/\overline{R}(\overline{u}_1\overline{u}_0+\overline{v}_1\overline{v}_0)}\; J_1(\overline{u}_1\,,\,\overline{v}_1\,;\,\sigma_1\,,\,\tau_1)\; d\overline{u}_1d\overline{v}_1 \quad ,$$

and

$$J_1(\overline{u}_1\,,\,\overline{v}_1\,;\,\sigma_1\,,\,\tau_1) = \iint D_1^{(2)}\left(\overline{u}_2\,,\,\overline{v}_2\,;\,\rho_1^{(2)}\right)\, e^{i/2(1/\overline{R}_1+1/\overline{R}_2-1/\overline{F}_2)\left(\overline{u}_2^2+\overline{v}_2^2\right)}$$

$$\times\, e^{-i[\overline{u}_2(\overline{u}_1/\overline{R}_1+\sigma_1/\overline{R}_2)+\overline{v}_2(\overline{v}_1/\overline{R}_1+\tau_1/\overline{R}_2)]}\; d\overline{u}_2d\overline{v}_2 \quad .$$

The scaling factors are now introduced through the equations

$$\overline{u}_0 = \alpha\overline{u}_0 \,, \overline{v}_0 = \alpha\overline{v}_0$$

$$\overline{u}_1 = \gamma u_1 \,, \overline{v}_1 = \gamma \overline{v}_1$$

$$\overline{u}_2 = \delta \overline{u}_2 \,, \overline{v}_2 = \delta \overline{v}_2$$

$$\sigma_1 = \beta\sigma \,, \tau_1 = \beta\tau \quad ,$$

so that one has

$$V(\beta\sigma \,, \beta\tau) = \frac{(-i)^3(\alpha\gamma\delta)^2}{(2\pi)^2\lambda_1} A_1 \frac{e^{i(\overline{R}_0+\overline{R}+\overline{R}_1+\overline{R}_2-\overline{F}_1-\overline{F}_2)}}{\overline{R}_0\overline{R}\,\overline{R}_1\overline{R}_2}$$

$$\times\, e^{i\beta^2/2\overline{R}_2(\sigma^2+\tau^2)} \iint \mathcal{J}_1(\alpha u_0 \,, \alpha v_0)\; e^{i\alpha^2/2(1/\overline{R}_0+1/\overline{R})\left(u_0^2+v_0^2\right)}$$

$$\times\, I_1(\alpha u_0 \,, \alpha v_0 \,;\, \beta\sigma \,, \beta\tau)\; du_0 dv_0 \quad ,$$

with

$$I_1(\alpha u_0 \,, \alpha v_0 \,;\, \beta\sigma \,, \beta\tau) = \iint D_1^{(1)}\left(\gamma u_1 \,, \gamma v_1 \,;\, \rho_1^{(1)}\right)$$

$$e^{i\gamma^2/2(1/\overline{R}+1/\overline{R}_1-1/\overline{F}_1)\left(u_1^2+v_1^2\right)}$$

$$\times\, e^{-i\alpha\gamma/\overline{R}(u_1u_0+v_1v_0)}\; J_1(\gamma u_1 \,, \gamma v_1 \,;\, \beta\sigma \,, \beta\tau)\; du_1 dv_1 \,,$$

and

$$J_1(\gamma u_1 \,, \gamma v_1 \,;\, \beta\sigma \,, \beta\tau) = \iint D_1^{(2)}\left(\delta u_2 \,, \delta v_2 \,;\, \rho_1^{(2)}\right)$$

$$e^{i\delta^2/2(1/\overline{R}_1+1/\overline{R}_2-1/\overline{F}_2)\left(u_2^2+v_2^2\right)}$$

$$\times\, e^{-i\delta[u_2(\gamma u_1/\overline{R}_1+\beta\sigma/\overline{R}_2)+v_2(\gamma v_2/\overline{R}_1+\beta\tau/\overline{R}_2)]}\; du_2 dv_2 \,.$$

From the equality of J and J_1, Rule 7 gives

$$\begin{cases} \dfrac{\beta\delta}{\overline{R}_2} = \dfrac{1}{\overline{r}_2} \\[2ex] \dfrac{\gamma\delta}{\overline{R}_1} = \dfrac{1}{\overline{r}_1} \\[2ex] \delta^2\left(\dfrac{1}{\overline{R}_1} + \dfrac{1}{\overline{R}_2} - \dfrac{1}{\overline{F}_2}\right) = \left(\dfrac{1}{\overline{r}_1} + \dfrac{1}{\overline{r}_2} - \dfrac{1}{\overline{f}_2}\right) \\[2ex] D_1^{(2)}\left(\delta u_2 , \delta v_2 ; \rho_1^{(2)}\right) = D^{(2)}\left(u_2 , v_2 ; \rho^{(2)}\right) \end{cases} .$$

Rule 7 applied to I and I_1 now gives

$$\begin{cases} \dfrac{\alpha\gamma}{\overline{R}} = \dfrac{1}{\overline{r}} \\[2ex] \gamma^2\left(\dfrac{1}{\overline{R}} + \dfrac{1}{\overline{R}_1} - \dfrac{1}{\overline{F}_1}\right) = \left(\dfrac{1}{\overline{r}} + \dfrac{1}{\overline{r}_1} - \dfrac{1}{\overline{f}_1}\right) \\[2ex] D_1^{(1)}\left(\gamma u_1 , \gamma v_1 ; \rho_1^{(1)}\right) = D^{(1)}\left(u_1 , v_1 ; \rho^{(1)}\right) \end{cases} ,$$

and the application to the remainder of the integrand gives

$$\begin{cases} \alpha^2\left(\dfrac{1}{\overline{R}_0} + \dfrac{1}{\overline{R}}\right) = \left(\dfrac{1}{\overline{r}_0} + \dfrac{1}{\overline{r}}\right) \\[2ex] \mathcal{J}_1(\alpha u_0 , \alpha v_0) = \mathcal{J}(u_0 , v_0) \end{cases} .$$

Finally, the external amplitude terms give

$$\frac{(\alpha\gamma\delta)^2}{\lambda_1} \frac{A_1}{\overline{R}_0 \overline{R}\, \overline{R}_1 \overline{R}_2} = \frac{1}{\lambda} \frac{A_0}{\overline{r}_0 \overline{r}\, \overline{r}_1 \overline{r}_2} .$$

The scaling equations for distances in the two systems are

$$\begin{cases} \alpha^2\left(\dfrac{1}{R_0} + \dfrac{1}{R}\right) = \dfrac{\lambda}{\lambda_1}\left(\dfrac{1}{r_0} + \dfrac{1}{r}\right) \\[2ex] \dfrac{\alpha\gamma}{R} = \dfrac{\lambda}{\lambda_1}\dfrac{1}{r} \\[2ex] \dfrac{\gamma\delta}{R_1} = \dfrac{\lambda}{\lambda_1}\dfrac{1}{r_1} \\[2ex] \dfrac{\beta\delta}{R_2} = \dfrac{\lambda}{\lambda_1}\dfrac{1}{r_2} \\[2ex] \gamma^2\left(\dfrac{1}{R} + \dfrac{1}{R_1} - \dfrac{1}{F_1}\right) = \dfrac{\lambda}{\lambda_1}\left(\dfrac{1}{r} + \dfrac{1}{r_1} - \dfrac{1}{f_1}\right) \\[2ex] \delta^2\left(\dfrac{1}{R_1} + \dfrac{1}{R_2} - \dfrac{1}{F_2}\right) = \dfrac{\lambda}{\lambda_1}\left(\dfrac{1}{r_1} + \dfrac{1}{r_2} - \dfrac{1}{f_2}\right) \end{cases} \quad .$$

Taking the lens apertures to be rect-functions, one finds the aperture scaling functions to be

$$\begin{cases} \rho_1^{(1)} = \dfrac{\gamma\lambda_1}{\lambda}\,\rho^{(1)} \\[2ex] \rho_1^{(2)} = \dfrac{\delta\lambda_1}{\lambda}\,\rho^{(2)} \end{cases} \quad .$$

The functional scaling relations are the same as before:

$$\begin{cases} U_1\left(\dfrac{\beta\lambda_1}{\lambda}\,\xi\,,\ \dfrac{\beta\lambda_1}{\lambda}\,\eta\right) = K\,U(\xi\,,\,\eta) \\[2ex] T_1\left(\dfrac{\alpha\lambda_1}{\lambda}\,x_0\,,\ \dfrac{\alpha\lambda_1}{\lambda}\,y_0\right) = T(x_0\,,\,y_0) \end{cases} \quad .$$

The amplitude scaling relation is again

$$A_1 = \frac{\lambda_1}{\lambda}\,\alpha\beta\gamma\,A_0\,\frac{r}{[\gamma r - (\alpha - \gamma)r_0]} \quad .$$

The general properties of these scaling equations are obviously similar to those for the one-lens system; therefore, no examples are needed.

VIII. GENERALIZATION TO AN n-LENS SYSTEM

There is a pattern to the scaling equations which allows one to generalize from this point to a system containing any number of lenses. This general result can be proved by induction starting with the diffraction integrals, but there would be very little virtue and a great deal of tedium in this.

An examination of the previous scaling equations shows that the scaling factors α and β occupy special positions in the hierarchy of scaling factors. This reflects their special relations to the input (target) and output (diffraction pattern) signals. The other scaling factors are "internal" to the system, each one being associated with a physical plane containing an aperture and a lens. For notational convenience, these internal scaling factors will be relabeled.

Let γ_{i-1} be associated with lens $\mathcal{L}_i$. Let α be the scaling factor for the target and β be the scaling factor for the diffraction pattern. Then, the scaling equations for distances in an n-lens system are:

$$\left\{\begin{array}{l}
\alpha\left(\frac{1}{R_0} + \frac{1}{R}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r_0} + \frac{1}{r}\right) \\[2ex]
\frac{\alpha\gamma_0}{R} = \frac{\lambda}{\lambda_1}\frac{1}{r} \\[2ex]
\frac{\gamma_{i-1}\gamma_i}{R_i} = \frac{\lambda}{\lambda_1}\frac{1}{r_i} \quad , \quad i = 1 \ldots n-1 \\[2ex]
\frac{\beta\gamma_{n-1}}{R_n} = \frac{\lambda}{\lambda_1}\frac{1}{r_n} \\[2ex]
\gamma_0^2\left(\frac{1}{R} + \frac{1}{R_1} - \frac{1}{F_1}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r} + \frac{1}{r_1} - \frac{1}{f_1}\right) \\[2ex]
\gamma_{i-1}^2\left(\frac{1}{R_{i-1}} + \frac{1}{R_i} - \frac{1}{F_i}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r_{i-1}} + \frac{1}{r_i} - \frac{1}{f_i}\right) \quad , \quad i = 2 \ldots n \quad .
\end{array}\right.$$

The aperture scaling relations are

$$D_1^{(i)}\left(\gamma_{i-1}u_i\,,\,\gamma_{i-1}v_i\,;\,\rho_1^{(i)}\right) = D^{(i)}\left(u_i\,,\,v_i\,;\,\rho^{(i)}\right)\ ,\quad i = 1\ \ldots\ n\quad ,$$

and if all the apertures are rect-functions, the aperture radii scale according to

$$\rho_1^{(i)} = \frac{\gamma_{i-1}\lambda_1}{\lambda}\,\rho^{(i)}\quad .$$

Finally, the functional equations and the amplitude equation are

$$\begin{cases} U_1\left(\dfrac{\beta\lambda_1}{\lambda}\,\xi\,,\,\dfrac{\beta\lambda_1}{\lambda}\,\eta\right) = K\,U(\xi\,,\,\eta) \\[2ex] T_1\left(\dfrac{\alpha\lambda_1}{\lambda}\,x_0\,,\,\dfrac{\alpha\lambda_1}{\lambda}\,y_0\right) = T(x_0\,,\,y_0) \\[2ex] A_1 = \dfrac{\lambda_1}{\lambda}\,\alpha\beta\gamma_0\,A_0\,\dfrac{r}{[\gamma_0 r - (\alpha - \gamma_0)r_0]}\ ,\quad n \geq 1\quad . \end{cases}$$

IX. TREATMENT OF A SPATIAL FILTER

A spatial filter is a common element in many coherent optical systems. The suggested design for the tactical optical pattern recognition system requires a spatial filter. It is quite simple to include spatial filters in the new optical scaling system, as will be seen.

A spatial filter is a physical object with specially chosen transmission properties. Mathematically, a spatial filter is represented by its transmission function. Because any real spatial filter has a finite aperture, it would be appropriate to call it an aperture (or aperture transmission) function, and to represent it by the symbol D (but not necessarily meaning a rect-function). This will be done. A spatial filter in the original system at plane m will be represented by $D^{(m)}(x_m, y_m)$ or by $D_s^{(m)}(x_m, y_m)$ if it is particularly important to distinguish it as a spatial filter.

The generality of the scaling equations will be increased if each spatial filter is treated as being associated with a lens, even though spatial filters often stand alone. Such a treatment allows for the possibility that a lens be used at the spatial filter in the model system, even if there is no lens at the spatial filter in the original system.

Using this approach, the inclusion of spatial filters in the formalism introduces no changes in the previously derived general scaling equations. Specifically, if the spatial filter is at the m^{th} lens plane, the contribution of this element to the scaling equations is of the form

$$\left\{\begin{aligned} &\frac{\gamma_{m-2}\,\gamma_{m-1}}{R_{m-1}} = \frac{\lambda}{\lambda_1}\frac{1}{r_{m-1}} \\ &\gamma_{m-1}^2\left(\frac{1}{R_{m-1}} + \frac{1}{R_m} - \frac{1}{F_m}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r_{m-1}} + \frac{1}{r_m} - \frac{1}{f_m}\right) \\ &D_{1S}^{(m)}\left(\frac{\gamma_{m-1}\,\lambda_1}{\lambda}\,x_m\,,\ \frac{\gamma_{m-1}\,\lambda_1}{\lambda}\,y_m\right) = D_S^{(m)}(x_m\,,\,y_m) \end{aligned}\right.$$

for an internal plane. The modifications required if the spatial filter is in either the first or the last lens plane are apparent from the general equations.

The distinguishing features of the spatial filter enter the calculations only when explicitly evaluating the preceding aperture-scaling equation. The general result is that in the model system the spatial filter transmission function is magnified by the factor

$$\frac{\gamma_{m-1}\,\lambda_1}{\lambda}\ ,$$

relative to the spatial filter in the original system. As an example, consider a spatial filter consisting of a central opaque spot of radius ρ_s. The spatial filter also has a finite total aperture, and is taken to be a circle of radius $\rho_m > \rho_s$, concentric with the spot. Then

$$D_s^{(m)}(x_m\,,\,y_m) = \mathrm{rect}\left[\frac{\left(x_m^2 + y_m^2\right)^{1/2}}{2\rho_m}\right] - \mathrm{rect}\left[\frac{\left(x_m^2 + y_m^2\right)^{1/2}}{2\rho_s}\right]\ .$$

Thus,

$$D_{1s}^{(m)}\left(\frac{\gamma_{m-1}\,\lambda_1}{\lambda}\,x_m\,,\ \frac{\gamma_{m-1}\,\lambda_1}{\lambda}\,y_m\right) = \mathrm{rect}\left[\frac{\gamma_{m-1}\,\lambda_1\left(x_m^2 + y_m^2\right)^{1/2}}{2\lambda\,\rho_{1m}}\right] - \mathrm{rect}\left[\frac{\gamma_{m-1}\,\lambda_1\left(x_m^2 + y_m^2\right)^{1/2}}{2\lambda\,\rho_{1s}}\right]\ ,$$

so that the scaling equation gives

$$\begin{cases} \rho_{1m} = \dfrac{\gamma_{m-1}\,\lambda_1}{\lambda}\,\rho_m \\[2ex] \rho_{1s} = \dfrac{\gamma_{m-1}\,\lambda_1}{\lambda}\,\rho_s \quad . \end{cases}$$

It is also instructive to examine spatial filter scaling in the case for which neither system has a lens at the spatial filter. One must then set $f_m = F_m = \infty$ and consider all equations that involve only the distances r_{m-1}, r_m, R_{m-1}, and R_m. These equations are now

$$\begin{cases} \dfrac{\gamma_{m-2}\,\gamma_{m-1}}{R_{m-1}} = \dfrac{\lambda}{\lambda_1}\,\dfrac{1}{r_{m-1}} \\[2ex] \gamma_{m-1}^2\left(\dfrac{1}{R_{m-1}} + \dfrac{1}{R_m}\right) = \dfrac{\lambda}{\lambda_1}\left(\dfrac{1}{r_{m-1}} + \dfrac{1}{r_m}\right) \\[2ex] \dfrac{\gamma_{m-1}\,\gamma_m}{R_m} = \dfrac{\lambda}{\lambda_1}\,\dfrac{1}{r_m} \quad . \end{cases}$$

Now, if the first and last equations are subtracted from the middle equation, the result is

$$\gamma_{m-1}\left[\frac{\gamma_{m-1} - \gamma_m}{R_m} + \frac{\gamma_{m-1} - \gamma_{m-2}}{R_{m-1}}\right] = 0 \quad .$$

This can be written as

$$\frac{\left(\dfrac{\gamma_{m-1}}{\gamma_m} - 1\right)}{r_m} + \frac{\left(\dfrac{\gamma_{m-1}}{\gamma_{m-2}} - 1\right)}{r_{m-1}} = 0 \quad .$$

This indicates that if the scaling factors γ_m, γ_{m-2} are considered to be at one's disposal, then γ_{m-1} is determined once the others are chosen. This is physically reasonable because it would be surprising if the scale on a spatial filter could be chosen completely independent of the scale of the rest of the optical system.

The intuitive choice (for no lenses at the spatial filters) is

$$\gamma_{m-1} = \gamma_m = \gamma_{m-2} \quad ,$$

which means that lenses m - 1, m + 1, and the spatial filter all scale from the original system by the same factor. This, however, is not the only scaling solution. Another scaling solution is given by

$$\gamma_{m-1}^2 = \left(\frac{1}{r_m} + \frac{1}{r_{m-1}}\right)\left(\frac{1}{\gamma_m r_m} + \frac{1}{\gamma_{m-2} r_{m-1}}\right)^{-1} \quad .$$

This reduces to the intuitive solution only for the choice $\gamma_m = \gamma_{m-2}$. This example shows, again, the considerable modeling flexibility available with this new system.

X. MODELING THE TACTICAL OPTICAL PATTERN RECOGNITION SYSTEM

The suggested design for the tactical optical pattern recognition system required four elements: two lenses followed by a spatial filter, which is, in turn, followed by the third lens. The system is shown in Figure 5. There is no lens at the spatial filter, so one must take $f_4 = \infty$. The spatial filter consists of a central obscuring spot, whose radius will be denoted by ρ_s. All the lenses have circular apertures.

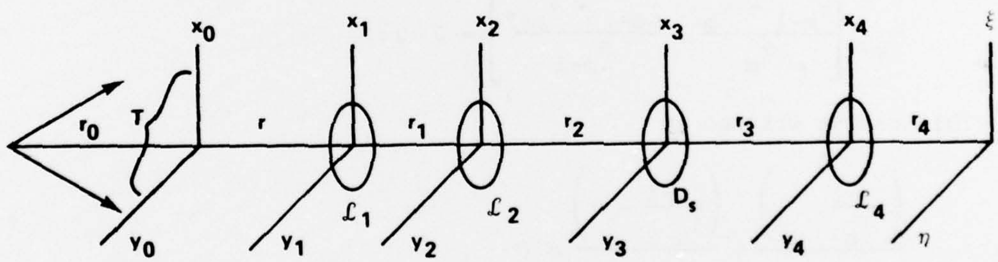


Figure 5. The tactical optical pattern recognition system.

The distance scaling equations are:

$$
\begin{cases}
\alpha\left(\frac{1}{R_0} + \frac{1}{R}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r_0} + \frac{1}{r}\right) \\
\frac{\alpha\gamma_0}{R} = \frac{\lambda}{\lambda_1}\frac{1}{r} \\
\frac{\gamma_0\gamma_1}{R_1} = \frac{\lambda}{\lambda_1}\frac{1}{r_1} \\
\frac{\gamma_1\gamma_2}{R_2} = \frac{\lambda}{\lambda_1}\frac{1}{r_2} \\
\frac{\gamma_2\gamma_3}{R_3} = \frac{\lambda}{\lambda_1}\frac{1}{r_3} \\
\frac{\beta\gamma_3}{R_4} = \frac{\lambda}{\lambda_1}\frac{1}{r_4} \\
\gamma_0^2\left(\frac{1}{R} + \frac{1}{R_1} - \frac{1}{F_1}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r} + \frac{1}{r_1} - \frac{1}{f_1}\right) \\
\gamma_1^2\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{F_2}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{1}{f_2}\right) \\
\gamma_2^2\left(\frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{F_3}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r_2} + \frac{1}{r_3}\right) \\
\gamma_3^2\left(\frac{1}{R_3} + \frac{1}{R_4} - \frac{1}{F_4}\right) = \frac{\lambda}{\lambda_1}\left(\frac{1}{r_3} + \frac{1}{r_4} - \frac{1}{f_4}\right)
\end{cases}
.
$$

The aperture scaling equations are:

$$
\begin{cases}
\rho_1^{(1)} = \dfrac{\gamma_0 \lambda_1}{\lambda} \rho^{(1)} \\[2ex]
\rho_1^{(2)} = \dfrac{\gamma_1 \lambda_1}{\lambda} \rho^{(2)} \\[2ex]
\rho_1^{(3)} = \dfrac{\gamma_2 \lambda_1}{\lambda} \rho^{(3)} \\[2ex]
\rho_{1s} = \dfrac{\gamma_2 \lambda_1}{\lambda} \rho_s \\[2ex]
\rho_1^{(4)} = \dfrac{\gamma_3 \lambda_1}{\lambda} \rho^{(4)}
\end{cases} .
$$

The functional and amplitude scaling equations are:

$$
\begin{cases}
U_1\left(\dfrac{\beta\lambda_1}{\lambda} \xi , \dfrac{\beta\lambda_1}{\lambda} \eta\right) = K\, U(\xi , \eta) \\[2ex]
T_1\left(\dfrac{\alpha\lambda_1}{\lambda} x_0 , \dfrac{\alpha\lambda_1}{\lambda} y_0\right) = T(x_0 , y_0) \\[2ex]
A_1 = \dfrac{\lambda_1}{\lambda} \alpha\beta\gamma_0\, A_0 \dfrac{r}{[\gamma_0 r - (\alpha - \gamma_0) r_0]}
\end{cases} .
$$

It is apparent that there is considerable design freedom (six scaling factors) available in specifying a laboratory model of the field-scale system. The determination of some particular models from these equations will be left for a subsequent report.

XI. DISCUSSION

The basic principles of a new system of optical scaling and modeling have been presented. These principles apply to any optical situation in which the illumination is totally coherent and for which the paraxial approximation is valid. The primary scaling relation is a geometrical similarity between the output diffraction patterns of an original optical system and its model. The optical scaling based on these principles is called ordinary scaling.

The theory of ordinary scaling was developed in detail for any system consisting entirely of lenses and apertures and spatial filters.

In particular, the work in this report was directed toward the scaling of optical pattern recognition systems that might be used tactically. Because of this emphasis, a special kind of target illumination was assumed in the derivations. This limitation is not serious, however, and its removal is simple. Thus, the theory developed here is essentially applicable to any optical pattern recognition or optical data processing system at any wavelength.

By using an example of a single-lens system, it was shown that this new theory of ordinary scaling makes it accurately possible to model very large scale systems with modestly-sized laboratory systems. The particular significance of this is that certain tactical parameters such as target signatures can be determined without the large expense of field tests using a field-scale system. The theory also allows a change in wavelength in the model, which can be another cost-saving convenience; it also allows a laboratory model using the field wavelength, obtaining a field-size diffraction pattern, and having the same power throughout, which means that field detectors and circuitry can be tested directly in the laboratory. The final development was a set of scaling equations applicable specifically to a previously specified tactical optical pattern recognition system.

As indicated briefly in Section I, the basic principles of ordinary scaling apply more broadly than the narrow development of this report might indicate. They apply, in fact, to any of optical phenomenon for which propagation effects can be expressed in a form like the Fresnel-Kirchhoff diffraction integral. It is the author's opinion that this includes the important phenomena of optical scattering by particle distributions, optical scattering by turbulent fluctuations, and optical path distortion by thermal effects.

It is the author's belief that the particular results presented in this report, and other results which can be developed from the basic principles of ordinary scaling are of considerable importance to Redstone Arsenal in areas outside of optical pattern recognition. The basis for the importance is money: ordinary scaling will allow laboratory measurements of field performance, where the only previous option was actual field measurement. It is the author's opinion that ordinary scaling can be used to design field-accurate laboratory measurements associated with the following problems:

a) Laser designation/guidance systems.

b) Smoke.

c) Turbulence.

d) Inclement weather.

e) Terrain/thermal effects.

f) Optical countermeasures.

DISTRIBUTION

Addressee	No. of Copies
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	12
Commander US Army Electronics Command Attn: DRSEL-CT-L, Dr. Buser	1
-VL-E, Mr. Flynn	1
-CT-L-A, Mr. Mirachi	1
-WL-D, Mr. Hardin	1
-WLN, Mr. Giambalvo	1
-WLS, Mr. Charlton Fort Monmouth, New Jersey 07703	1
Commander US Army Armaments Command Attn: SARRI-LR, Dr. Amoruso	1
-T-SPA, Mr. Fosbinder	1
DRCPM-CAWS, COL S. T. Post, Jr. Rock Island, Illinois 61201	1
Commander US Army Materiel Command Attn: DRCRD-MT 5001 Eisenhower Avenue Alexandria, Virginia 22314	5
Director US Army Materiel Systems Analysis Agency Attn: DRXSY-D	1
-CS, Mr. Marchetti Aberdeen Proving Ground, Maryland 21005	1
Commander Edgewood Arsenal Attn: SAREA-TD	1
-DE-DD, Mr. Tannebaum Aberdeen Proving Ground, Maryland 21010	1
Director US Army Air Mobility Research and Development Laboratory Eustis Directorate Attn: SAVDL-EU-MO, Mr. Joe Ladd Fort Dustis, Virginia 23604	1
Director Night Vision Laboratory (USAECOM) Attn: DRSEL-NV-VI, Mr. Rodak Fort Belvoir, Virginia 22060	1
Commander US Army Mobility Equipment Research and Development Center Attn: STSFB-BA, Mr. Atkinson Fort Belvoir, Virginia 22060	1
Director Atmospheric Sciences Laboratory (USAECOM) Attn: DRSEL-BL, Mr. Lindberg White Sands Missile Range, NewMexico 88002	1
Commander Picatinny Arsenal Attn: SMUPA-AD-C, Mr. Heinemann	1
FRL, PYRO, Mr. Novak Dover, New Jersey 07801	1
Commander Frankford Arsenal Attn: SARFA-L5000, Dr. McNeill Philadelphia, Pennsylvania 19137	1
Library US Army War College Carlisle Barracks, Pennsylvania 17013	

Addressee	No. of Copies
Director Benet Weapons Laboratory Watervliet Arsenal Attn: SWEWV-RDD-SE, Mr. Andrade Watervliet, New York 12189	1
Chief, Missile EW Technical Area US Army Electronic Warfare Laboratory Attn: ANSEL-WL-M	1
-MJ, Mr. Kasparek	1
-WL-MS, Sylvia Hoihjelle White Sands Missile Range, New Mexico 88002	1
Director, US Army Materials and Mechanics Research Center Attn: DRXMR-HH, Mr. Dignam Watertown, Massachusetts 02172	1
Director, Harry Diamond Laboratories Attn: DRXDO-RCB, Dr. Gleason	1
-SAB, Mr. Johnson Connecticut Ave & Van Ness St., NW Washington, DC 20438	1
Commander US Marine Corps Development and Education Command Attn: Development Center Quantico, Virginia 22314	1
Commander US Marine Corps, Code AX Washington, DC 20380	1
Director Waterways Experiment Station Corps of Engineers Attn: Mr. Grabau	1
Dr. LaGarde P.O. Box 631 Vicksburg, Mississippi 39180	1
Commander US Army Infantry School Attn: ATSH-I Fort Benning, Georgia 31905	1
Commander US Army Armor School Attn: ATSB-CD-M Fort Knox, Kentucky 40121	1
Commander US Army Air Defense School Air Defense Agency Attn: ATSA-CTD-MS Fort Bliss, Texas 79916	1
Commander US Army Ordnance Center & School Attn: ATSL-CTD-MS-R (LT Roberts) Aberdeen Proving Ground, Maryland 21005	1
Commander US Army Cobined Arms Combat Development Activity Attn: ATCACC-TH	2
-E, Mr. Bray Fort Leavenworth, Kansas 66027	1
Commander US Army Combat Development Experimentation Command Attn: Tech Library - Box 22 Fort Ord, California 93941	1

	No. of Copies
Headquarters	
US Army Foreign Science	
and Technology Center	
Attn: DRXSP-CB1, Mr. Pearson	2
220 Seventh Street, NE	
Charlottesville, Virginia 22901	
Headquarters	
Department of the Army	
The Pentagon	
Attn: DARD-DDM, MAJ Kopsac	5
Washington, DC 20310	
AFATL/DLWS	
Attn: MAJ D. Couture	1
Eglin AFB, Florida 32542	
Defense Advanced Research Project Agency	
Tactical Technology Office	
Attn: Dr. C. H. Church	1
1400 Wilson Boulevard	
Arlington, Virginia 22209	
Commander	
Maverick Systems Project Office	
Attn: ASD/SD 65M	1
65-ENYX	1
Wright-Patterson Air Force Base, Ohio 45433	
Commander	
Naval Weapons Center	
Attn: Mr. C. P. Smith (Code 405)	1
Mr. L. W. Nichols (Code 60401)	1
China Lake, California 93555	
Office of Assistant for Land Warfare	
Director for Defense Research	
and Engineering	
Attn: Dr. Richard E. Schwartz	1
Pentagon, Room 3E1025	
Washington, DC 20301	
Naval Air Systems Command	
Attn: LT CMD F. G. Woods	1
AIR-5109C, Room 1258JP2	
Washington, DC 20360	
Headquarters, USAF (RDPA)	
Director of Dev & Acq, DCS/R&D	
Attn: LTC Bill Seufert	1
Washington, DC 20330	
Headquarters, Air Force Systems Command	
Attn: AFSC DLCAW/Mr. R. Cross	1
Andrews Air Force Base	
Washington, DC 20334	
Air Force Special Communications Center	
Technical Sensors Division	1
San Antonio, Texas 78243	
Director	
US Army Ballistic Research Laboratories	
Attn: DRXBR-CA (Mr. H. L. Reed)	1
-CAL (Mr. Arthur LaGrange)	1
DRXSY, Mr. T. Dolce	1
Aberdeen Proving Ground, Maryland 21005	

	No. of Copies
Director	
US Army Human Engineering Laboratory	
Attn: DRXHE (Mr. Richard R. Kramer)	1
Aberdeen Proving Ground, Maryland 21005	
Commander	
Headquarters, US Army Training	
and Doctrine Command	
Attn: DCS-CD (ATCD-CS-PL)	1
(ATCD-CS-FT) LTC Davis	1
Fort Monroe, Virginia 23651	
Headquarters MASSTER	
Attn: AFMAS-CSS (Mr. Kirkwood)	1
-AC-PE3 (MAJ Alexander)	1
Fort Hood, Texas 76544	
Commander	
Naval Weapons Laboratory	
Attn: Mr. George Williams, Code EJP	1
Mr. William Spicer, Code EJM	1
Dahlgren, Virginia 22448	
Commander	
US Army Field Artillery School	
Attn: ATSF-CTD-WS	1
-S	1
Fort Sill, Oklahoma 73503	
DRSMI-FR, Mr. Strickland	1
-LP, Mr. Voigt	1
-V, MAJ Harborth	1
-TRADOC-LNO	1
-YDR, Mr. Todd	1
-D	1
-QR, Mr. G. J. Hutcheson	1
-R, Dr. McDaniel	1
Dr. Kobler	1
-RBD	3
-RD	1
-RE	1
-REO, Mr. Sitton	1
Mr. Widenhofer	1
Mr. Farmer	1
Mr. Anderson	10
-RFL	1
-RFA	1
-RFW	1
-RFD	1
-RFS	1
-RFT	1
-RFE	1
-RFGA	1
-RFGB	1
-RG	1
-RL	1
-RT	1
-RPR (Record Copy)	1
(Reference Copy)	1
-RE, Mr. Pittman	1